Notes on Exponents and Logarithms

Exponents

Essential for interest rate calculations
Denotes repeated multiplication of same quantity

EXPONENT

$$x^t = \underbrace{x \cdot x \cdot x \cdot \ldots \cdot x}_{t\ times}$$

BASE

Formula: Exponents

Products	$b^c \cdot b^d = b^{c+d}$
Powers	$(b^c)^d = (b^d)^c = b^{c \cdot d}$
Negative exponent	$b^{-c} = \frac{1}{b^c}$
Quotients	$\frac{b^c}{b^d} = b^{c-d}$
Zero Power	$b^0 = 1$
Roots	$\sqrt[n]{b} = b^{1/n}$

Formula: Interest Rates

- **Simple Interest**
  Total Interest over n periods: $I = P * r * n$
- **Compound Interest**
  - compounding annually:
  $$A = P(1+r)^t$$
  - compounding for n periods per year
  $$A = P\left(1+\frac{r}{n}\right)^{tn}$$

A = final amount, P = present amount, r = interest rate, t = number of years, n = number of periods per year

Logarithmic Function

The inverse of an exponential function is called a **Logarithmic Function**

Number = base$^{\text{power}}$

Log_{base} (Number) = power

Logarithmic form		*Exponential form*
$y = \log_b x$	$\Leftrightarrow$	$x = b^y$

Formula: Logs

$$y = \log_b x \qquad \Leftrightarrow \qquad x = b^y$$

$$\log_b\left(b^x\right) = x \qquad b^{\log_b(x)} = x$$

$$\log_b(c \cdot d) = \log_b c + \log_b d$$

$$\log_b\left(\frac{c}{d}\right) = \log_b c - \log_b d$$

$$\log_b c^d = d \cdot \log_b c$$

$$\log_b b = 1 \qquad \log_b 1 = 0$$

$$\log_b(c + d) = \log_b(c + d)$$

Logarithmic Function

Special case

•Base = e = *2.71828* (natural log = ln)

(in honour of Scottish mathematician Napier)

$$y = \ln x = \log_e x \qquad \Rightarrow \ln e^a = a$$

$$\Rightarrow e^{\ln b} = b$$

Formula: Exponential and Logarithmic equations

$$e^a = e^b \Leftrightarrow a = b$$

$$\ln a = \ln b \Leftrightarrow a = b$$

$$e^{\ln b} = b \qquad \ln e^b = b$$