MBA Business Foundations, Quantitative Methods: Session One

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About me

- Profession of Decision Sciences
- My research is in Bayesian statistics, ethics of AI/ML
- Postdoc, California Institute of Technology
- PhD/MS, University of Michigan (Ann Arbor)
- Former trial lawyer
- Teach MBA Management Decision Making and PhD Bayesian Stats

Course structure

- Two overarching features: (a) mixed backgrounds, (b) busy schedule
- Structure: follow a clear path + bonus adventures for the curious

  Ex: I will post all my workflow (LaTeX, Python, Mathematica notebooks)
- Focus on exercises/learning by doing!
- 5 classes, focus on applications to management and finance
- Readings before each lecture
- Exercises after each lecture (due the following lecture)

  Will not be graded, but I will post solutions
- Study period in the afternoon, I will be around (Office 0.09)
- If anything is unclear, come talk to me!
- Website: borisbabic.com/teaching/inseadqm/home

Content

Basics	Functions Linear Inverse Two equations Quadratic
Exponents	Exponents Application: interest rates Exponential functions Logarithmic functions
Logarithms	Logarithmic functions Logarithmic and exponential equations Case: pricing Derivatives
Derivatives	Optimal decisions Case: production Statistics
Uncertainty	Probability & statistics Normal distribution

Today

Basics	Functions Linear Inverse Two equations Quadratic
Exponents	Exponents Application: interest rates Exponential functions Logarithmic functions
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Constants vs. variables

Constant

- Definition: placeholder for a given or fixed value
- Notation: a, b, c
- Examples:
  - Maximum number of units that can be produced on a production line
  - Height of the Eiffel tower

Variable

- Definition: Placeholder for an unknown value
- Notation: x, y, z
- Examples:
  - Number of units produced each day on a production line
  - Height of a student in this class

Continuous vs. discrete variables

Continuous

- Can take values within a range
- Examples: height, weight, etc.

Discrete

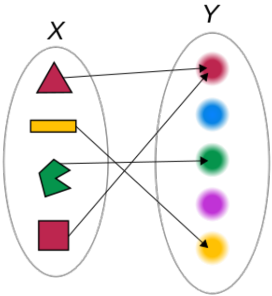
- Can take only certain values (typically whole numbers)
- Examples: number of children, number of defective products, number of weeks worked

Functions

- A function is a type of map:

$$x \text{ (Input)} \longrightarrow f \text{ (Function)} \longrightarrow y = f(x)\text{(Output)}$$

- Here we say f maps x to y. For example, the following function maps shapes to their associated colors.



- Does it matter that no blue shape? That two red shapes?
- x is the independent variable, y is the dependent variable.
- f is the operation done on x to get y – the function, usually denoted f, g, h.

Functions: examples

- Eg: Let $f(x) = x + 2$. Then if $x = 3$, $y = f(x) = 5$.
- Eg: Amount of interest earned (I) depends on the length of time money is invested (t), given both money invested (p) and interest rate (r):

  $I = t \times p \times r$

  $I = 10000 \times 0.04t = 400t$. If $t = 5$ then $I = \$2,000$

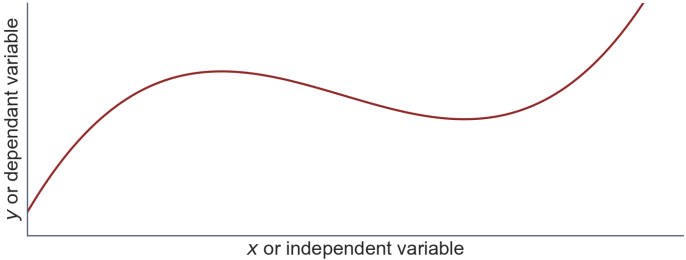
  $y = f(t)$
- Eg: Revenue of a firm (R) is a function of quantity of product sold (q), given the price (p)

  $R = \text{price} \times \text{quantity} \rightarrow R = p \times q \rightarrow R = 5q$

  $R = g(q)$ (why does p not appear in the expression?)

Graphs

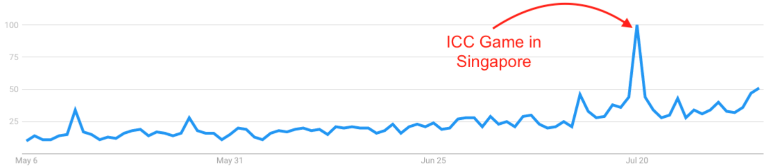
A convenient way to visualize functions:



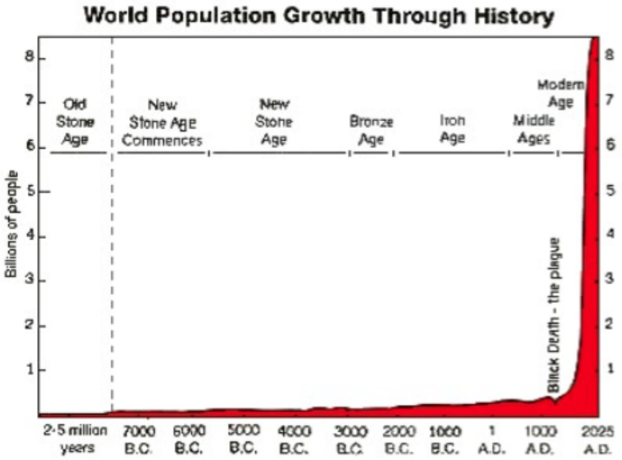
Gives a visual representation of the relationship between two quantities

Some examples of graphs

Google searches for "Manchester United" in Singapore as a function of time (previous 90 days)



Some examples of graphs

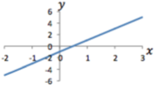
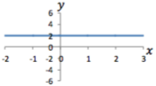
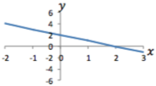


Linear functions

Functions of a special form:

$$y = ax + b$$

Slope (→ a) y-axis intercept (→ b)

	CRITERIA	EXAMPLE	GRAPH
INCREASING	$a > 0$	$y = 2x - 1$	
CONSTANT	$a = 0$	$y = 2$	
DECREASING	$a < 0$	$y = -x + 2$	

Linear functions

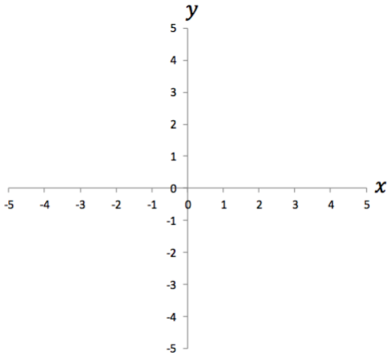
How to plot a linear function $y = ax + b$?

First, find two points:
.... easiest: those crossing axis

crossing y-axis: $(0, b)$

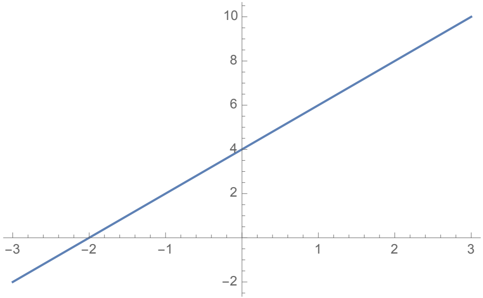
crossing x-axis: $\left(-\frac{b}{a}, 0\right)$

Second, draw line between and beyond



Example

- Ex: Let $f(x) = 2x + 4$. Plot this graph.
- $(0, b) = (0, 4)$
- $(-b/a, 0) = (-4/2, 0) = (-2, 0)$



Finding the linear form

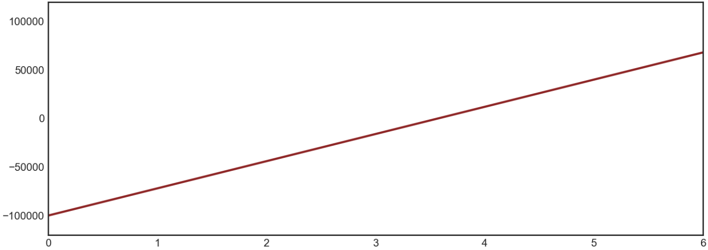
A grocery store owner starts her business with debts \$100,000. After operating for five years, she has accumulated a net profit of \$40,000. Write a linear rule for profit as a function of time. That is, write it in the form

$$y = ax + b$$

where y is profit and x is time.

Finding the linear form

$$y = -100000 + 28000x$$



Linear functions are...

- Easy to estimate
- Easy to analyze
- Easy to interpret (and surprisingly general!)

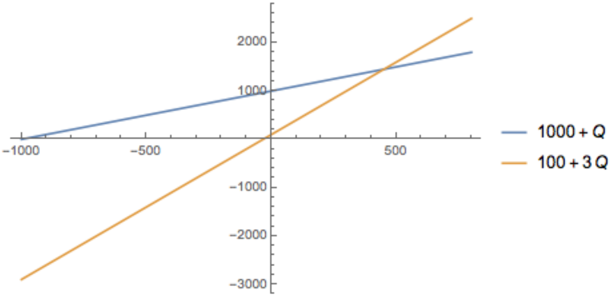
Finding the intersection of two lines

- Example: Nuclear vs. fuel power plants
- Suppose cost C is a linear function of quantity Q, where N stands for Nuclear and F stands for Fuel.

  $C_N = 1000 + Q_N$

  $C_F = 100 + 3Q_F$
- Plot the two lines
- At what point do the two plants have the same cost?

Finding the intersection of two lines



Inverse functions

An inverse function is a different type of map:

$$x = f^{-1}(y) \text{ (Input)} \longleftarrow f^{-1} \text{ (Inverse function)} \longleftarrow y \text{ (Output)}$$

- Note that $f^{-1}(f(x)) = x$
- Ex: if $f(x) = x^2$, what is $f^{-1}(x)$?
- Ex: If $g(x) = x^3 + 3$, what is $g^{-1}(x)$?
- Ex: if $h(x) = 7x^2 + 4$ what is $h^{-1}(x)$?

→ Answers:

- $f^{-1}(x) = \sqrt{x}$
- $g^{-1}(x) = \sqrt[3]{x-3}$
- $h^{-1}(x) = \sqrt{\frac{x-4}{7}}$

Recipe

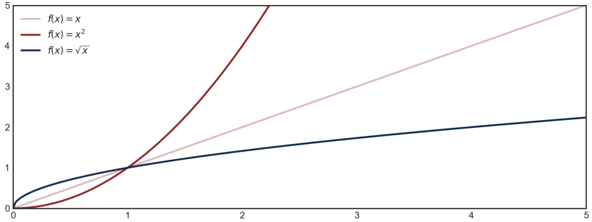
1. Replace $f(x)$ with a y
2. Swap x and y
3. Solve for y
4. Replace y with f^{-1}

Example from above:

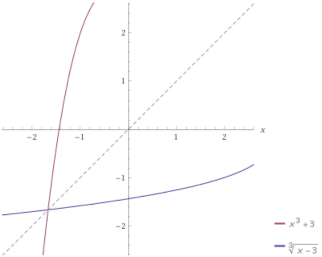
$$g(x) = x^3 + 3 \quad \text{(original function)}$$
$$\leftrightarrow y = x^3 + 3 \quad \text{(step 1)}$$
$$\leftrightarrow x = y^3 + 3 \quad \text{(step 2)}$$
$$\leftrightarrow y = \sqrt[3]{x - 3} \quad \text{(step 3)}$$
$$\leftrightarrow g^{-1}(x) = \sqrt[3]{x - 3} \quad \text{(step 4)}$$

Graphical relationship

$\sqrt{x^2} = x$



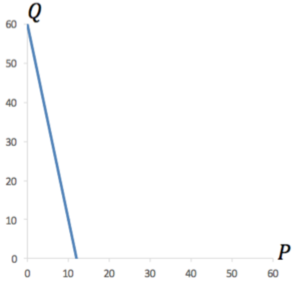
$\sqrt[3]{x^3 + 3 - 3} = x$



Example from economics

DEMAND FUNCTION

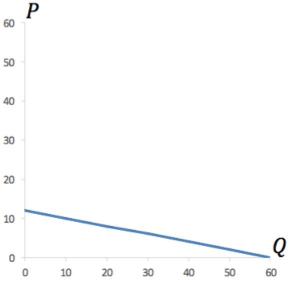
$Q = 60 - 5P$



INVERSE DEMAND FUNCTION

$5P = 60 - Q$

$\Rightarrow P = 12 - \frac{Q}{5}$



Systems of equations

	By substitution	By elimination
Method	Find x in the first equation, plug it into the second equation	Eliminate one unknown by adding up the two equations
Examples	$3x - 2y = 16$ $x + y = 2$	$x + y = 7$ $x - y = 1$

- By substitution (left panel example):

$$\begin{aligned} x = 2 - y &\rightarrow 3(2-y) - 2y = 16 \\ &\rightarrow 6 - 3y - 2y = 16 \\ &\rightarrow 6 - 5y = 16 \\ &\rightarrow 5y = -10 \\ &\rightarrow y = -2 \rightarrow x = 4 \end{aligned}$$

- By elimination (right panel example): $2x = 8 \rightarrow x = 4 \rightarrow y = 3$

Examples

- Ex 1:

  $3x - y = 7$

  $2x + 3y = 1$
- Ex 2:

  $5x + 4y = 1$

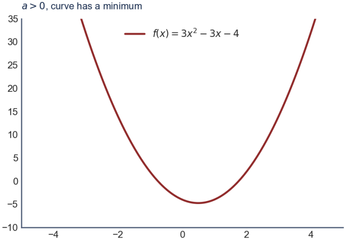
  $3x - 6y = 2$
- Solution 1: $x = 2, y = -1$
- Solution 2: $x = 1/3, y = -1/6$

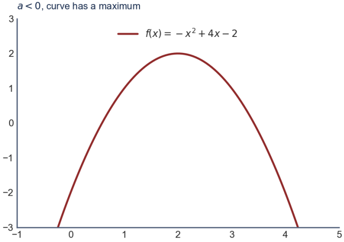
Quadratic functions

Another special type of function (a type of polynomial), of the form

$$ax^2 + bx + c$$

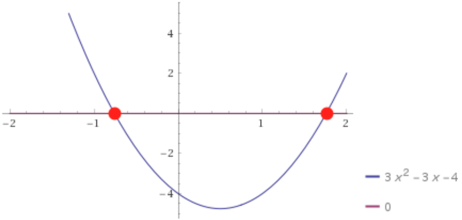
- When $a = 0$ we recover a linear function.
- When $a \neq 0$, this is a nonlinear function. Its graph is a continuous curve called a parabola.





Quadratic equations

- Solving quadratic function equal to 0.
- Goal: x such that $ax^2 + bx + c = 0$.



- Corresponds to the intersection(s) of the curve with $f(x) = 0$ line.
- Will there aways be solutions to this problem?
- Depends on the value of $b^2 - 4ac$.

Quadratic equations

- In general, when $ax^2 + bx + c = 0$, the roots are:

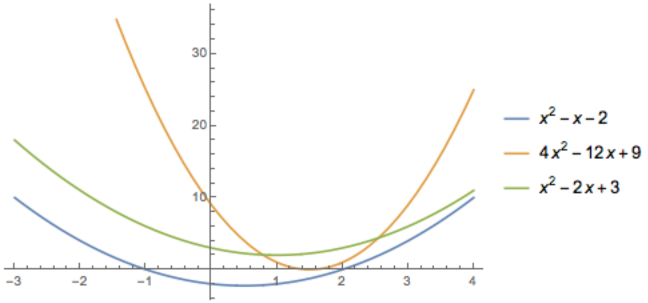
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- If $b^2 - 4ac > 0$ then 2 roots
- If $b^2 - 4ac = 0$ then 1 root
- If $b^2 - 4ac < 0$ then no roots

Exercises

- Ex 1: Solve $x^2 - x - 2 = 0$
- Ex 2: Solve $4x^2 - 12x + 9 = 0$
- Ex 3: Solve $x^2 - 2x + 3 = 0$
- Solution 1: $x = -1, x = 2$
- Solution 2: $x = 3/2$
- Solution 3: No real solution

Graphed solutions



Application to market equilibrium

Suppose that supply, S, and demand, D, for a product are functions of the product price, p:

$S = p^2 + 10p + 10$

$D = 110 - 10p$

At what price will supply equal demand?

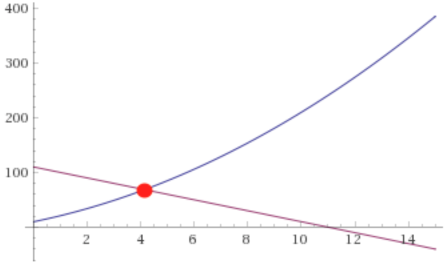
Application to market equilibrium

$$p^2 + 10p + 10 = 110 - 10p$$

$$\leftrightarrow p^2 + 20p - 100 = 0$$

$$\rightarrow p = \frac{-20 \pm \sqrt{20^2 - 4 \times 1 \times -100}}{2 \times 1}$$

$$p \approx 4.24$$



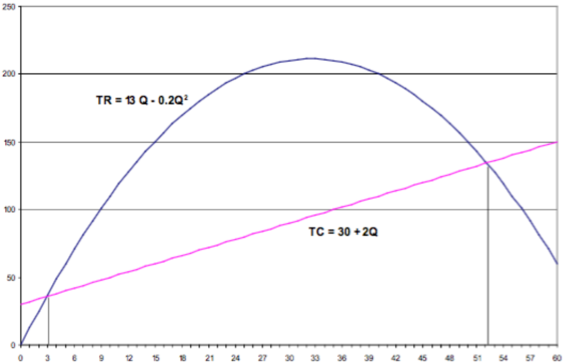
Profit-break even analysis

The demand function for a good is given as $Q = 65 - 5p$, where Q is quantity and p is price. Fixed costs are $\$30$ and each unit produced costs an additional $\$2$.

Write down the equations for total revenue and total costs as a function of Q.

Find the break-even point(s).

Application to market equilibrium



Resources

- Paul's Notes (for excellent notes): http://tutorial.math.lamar.edu/Extras/AlgebraTrigReview/AlgebraTrigIntro.aspx
- Khan Academy Algebra (for additional lectures): https://www.khanacademy.org/math/algebra
- WolframAlpha (for computing answers): https://www.wolframalpha.com/
- Math Stack Exchange (for questions): https://math.stackexchange.com/

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